

Article Title

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Abstract. The abstract serves both as a general introduction to the topic and as a brief, non-technical summary of the main results and their implications. Authors are advised to check the author instructions for the journal to which they are submitting for word limits and to determine whether structural elements such as subheadings, citations, or equations are permitted.

Keywords: Keyword1, Keyword2, Keyword3, Keyword4

1 Introduction

The Introduction section, of referenced text Manna et al. [1] expands on the background of the work (some overlap with the Abstract is acceptable). The introduction should not include subheadings (cf. Belfield et al. [2]).

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2 Theoretical background

Consider a semi-infinite medium covered by a FRV layer of thickness (H), as illustrated in Figure 1. The material particles are polarized in x -axis direction, ‘ O ’ is the intersecting point of topmost layer (m_1) and a substrate (m_2). Love wave propagation is assumed to be horizontally polarized along the x -axis, while the z -axis is vertically downward. Therefore, the total out-of-plane displacement components (u_1, u_2, u_3) along the (x, y, z) direction, respectively, can be expressed as

$$u_1 = u_3 = 0, \quad u_2 = u_2(x, z, t), \quad \text{and} \quad \frac{\partial}{\partial y} \equiv 0.$$

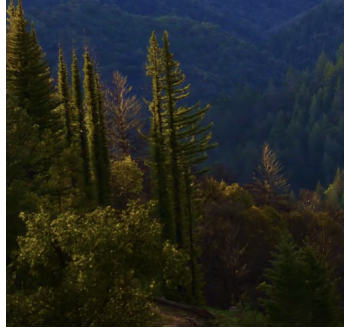


Fig. 1: Geometry of the present model

3 Equations

3.1 Solution

The governing equation of motion for the medium (m_1) are given by Belfield et al. [2]

$$\frac{\partial \sigma_{21}^{m_1}}{\partial x} + \frac{\partial \sigma_{23}^{m_1}}{\partial z} = \rho_1^{m_1} \frac{\partial^2 u_2^{m_1}}{\partial t^2}, \quad (1)$$

where, $\sigma_{21}^{m_1}$, $\sigma_{23}^{m_1}$ are the stress tensor components, $\rho_1^{m_1}$ represents the density of medium m_1 .

3.2 Results

The governing equation of motion of the medium (m_2) are given by [2]

$$(\lambda_2^{m_2} + \mu_2^{m_2} + \eta_2^{m_2} \nabla^2) \nabla (\nabla \cdot \mathbf{u}_r^{m_2}) + (\mu_2^{m_2} - \eta_2^{m_2} \nabla^2) \nabla^2 \mathbf{u}_r^{m_2} = \rho_2^{m_2} \frac{\partial^2 \mathbf{u}_r^{m_2}}{\partial t^2}, \quad (2)$$

where, $\lambda_2^{m_2}$ and $\mu_2^{m_2}$ are the Lamé constants and $\nabla^2 \equiv \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right)$ is the two-dimensional Laplacian operator, $\eta_2^{m_2} = \mu_2^{m_2} l^2$ is the coupled stress coefficient, l is the characteristic length, $\mathbf{u}_r^{m_2}$ is the displacement vector and $\rho_2^{m_2}$ is the mass density of the intermediate layer. With the help of the aforementioned conditions, we obtain $\nabla \cdot \mathbf{u}_r^{m_2} = 0$, and Eq. (9) can be written as

$$(\mu_2^{m_2} - \eta_2^{m_2} \nabla^2) \nabla^2 \mathbf{u}_r^{m_2} = \rho_2^{m_2} \frac{\partial^2 \mathbf{u}_r^{m_2}}{\partial t^2},$$

$$\left(\frac{\partial^2 u_2^{m_2}}{\partial x^2} + \frac{\partial^2 u_2^{m_2}}{\partial z^2} \right) - l^2 \left(\frac{\partial^4 u_2^{m_2}}{\partial x^4} + \frac{\partial^4 u_2^{m_2}}{\partial z^4} + 2 \frac{\partial^4 u_2^{m_2}}{\partial x^2 \partial z^2} \right) = \frac{1}{c_2^2} \frac{\partial^2 u_2^{m_2}}{\partial t^2}, \quad (3)$$

where, $c_2 = \sqrt{\frac{\mu_2^{m_2}}{\rho_2^{m_2}}}$. Assuming that $u_2^{m_2}$ be the solution of Eq. (10) is

$$u_2^{m_2}(x, z, t) = U_2^{m_2}(z) e^{i(kx - \omega t)}, \quad (4)$$

where, ω and k describe angular frequency and wavenumber, respectively. Substituting u_2^{m2} in Eq. (10), we obtain

$$\frac{d^4 U_2^{\text{m2}}}{dz^4} - P \frac{d^2 U_2^{\text{m2}}}{dz^2} + Q U_2^{\text{m2}} = 0, \quad (5)$$

where, $P = 2k^2 + \frac{1}{l^2}$ and $Q = k^4 - \frac{k^2}{l^2} \left(\frac{c^2}{c_2} - 1 \right)$. Thus, $U_2^{\text{m2}}(z)$ can be written as a solution of Eq. (12):

$$U_2^{\text{m2}}(z) = T_3 e^{-a_1 z} + T_4 e^{-a_2 z}, \quad (6)$$

where, $a_1 = \sqrt{\frac{P + \sqrt{P^2 - 4Q}}{2}}$, $a_2 = \sqrt{\frac{P - \sqrt{P^2 - 4Q}}{2}}$ and T_3, T_4 are arbitrary constants.

We obtain the final solution for the half-space using Eq. (13) in Eq. (11).

$$u_2^{\text{m2}}(x, z, t) = (T_3 e^{-a_1 z} + T_4 e^{-a_2 z}) e^{i(kx - \omega t)}. \quad (7)$$

For a coupled stress layer, the constitutive equations are as follows:

$$\begin{aligned} \sigma_{jr}^{\text{m2}} &= \lambda_2^{\text{m2}} e_{kk} \delta_{rj} + 2\mu_2^{\text{m2}} e_{rj} - 2\eta_2^{\text{m2}} \nabla^2 \omega_{rj}, \\ &= \lambda_2^{\text{m2}} u_{k,k}^{\text{m2}} \delta_{rj} + \mu_2^{\text{m2}} (u_{r,j}^{\text{m2}} + u_{j,r}^{\text{m2}}) - \eta_2^{\text{m2}} \nabla^2 (u_{r,j}^{\text{m2}} - u_{j,r}^{\text{m2}}), \end{aligned} \quad (8)$$

where, σ_{jr}^{m2} represents non-symmetric force stress-tensor, u_{rj}^{m2} represents the displacement component, δ_{rj} represents the Kronecker's delta, $e_{rj} = \frac{1}{2}(u_{r,j}^{\text{m2}} + u_{j,r}^{\text{m2}})$ represents the symmetric stress tensor, $\omega_{rj} = \frac{1}{2}(u_{r,j}^{\text{m2}} - u_{j,r}^{\text{m2}})$ (skew-symmetric rotation stress tensor).

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Acknowledgements are not compulsory. Where included they should be brief. Grant or contribution numbers may be acknowledged. Please refer to Journal-level guidance for any specific requirements.

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68 Appendix

$$Y_3 = \left(-\frac{a_1}{2}\overline{\mu_r} + (\overline{\mu_s} - \overline{\mu_r}) \left(\ell_1 \ell_3 i k - \frac{a_1}{2} \ell_3^2 \right) \right), \quad Y_4 = \left(\theta_1 \overline{\mu_r} + (\overline{\mu_s} - \overline{\mu_r}) \theta_1 \ell_3^2 \right), \\ Y_5 = \left(a_2 \mu_2^{\text{m}2} - a_2 \mu_2^{\text{m}2} k^2 l^2 + a_2^3 \mu_2^{\text{m}2} l^2 \right), \quad Y_6 = \left(a_2 \mu_2^{\text{m}2} - a_2 \mu_2^{\text{m}2} k^2 l^2 + a_2^3 \mu_2^{\text{m}2} l^2 \right), \\ Y_7 = \left(-\frac{a_1}{2}\overline{\mu_r} + (\overline{\mu_s} - \overline{\mu_r}) \left(\ell_1 \ell_3 i k - \frac{a_1}{2} \ell_3^2 \right) + \kappa_n \right), \quad Y_8 = \left(\theta_1 \overline{\mu_r} + (\overline{\mu_s} - \overline{\mu_r}) \theta_1 \ell_3^2 \right).$$

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